## RAGLAN MATHS EVENING PART 2

THE
PRACTICAL SESSION

## THIS EVENING

- We will:
- Explore how manipulatives are integrated into our curriculum to enhance learning experiences across Key Stage I and Key Stage 2.
- Engage in hands-on activities that demonstrate the effectiveness of manipulatives in teaching various mathematical concepts.
- Learn strategies for supporting your child's mathematical development at home using everyday materials.
- Manipulatives are not only for KSI!


## WHAT ARE MANIPULATIVES?

- The practical maths apparatus that is used in classrooms such as multilink cubes, Dienes, counters and bead strings. They are equipment that children can pick up and manipulate and which can help children to understand the relationship between numbers and the number system.
- They are objects which can be manipulated in real life. You can grab them with your hands. In maths, we use manipulatives as concrete objects that allow learners to be shown concepts or ideas in a hands-on format. Manipulatives can be made from almost any material.
- Importance of these has been known for a long time: Jean Piaget's (I95I) work which suggested that children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources


## A LITTLE HISTORY

- The history of the use of manipulatives in the classroom goes back over fifty years. A succinct historical summary of this is offered by Patricia Moyer (200I). She comments on Jean Piaget's (195I) work which suggested that children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. This was later built on by Zoltan Dienes (1969) who developed his base apparatus, and Caleb Gattegno and Georges Cuisenaire (1954) with their development of Cuisenaire rods.


## TASK I

Look at the range of manipulatives on your tables.
Discuss and record on the paper:

- What you think they could be used for
- Which have you used/seen before
- Which year group do you think they would be most useful


## TALK

- When solving the problems, please do share your thoughts with those around you.
- A significant part of the mastery concept is about children being to articulate their knowledge and understanding.
- Being able to explain WHY something is or is not the answer is a vital skill as children move through their maths journey.


## DOUBLE SIDED COUNTERS

- This is one of the most diverse manipulatives to use and a useful resource for any classroom from EYFS to Key Stage 3.
- 2-sided counters are a great support for so many different maths topics across all year groups including: place value, written calculations, fractions, decimals, percentages, ratio and algebra.



## TENS FRAMES



- A tens frame is used to represent numbers within and up to ten. Children can compose and decompose each number in a variety of ways which helps them develop number sense and to understand different visual representations of numbers.
- Usually, tens frames are used with manipulative resources but they can also be used pictorially on a worksheet or using a laminated frame and whiteboard pen or even in a lesson presentation slide.


## DIENES



- Dienes - also known as base ten - are versatile manipulatives, which allow teachers and students to represent and understand numbers.
- They can be used to teach a large proportion of the maths curriculum including place value, calculation and decimals throughout the primary age range.


## MULTILINK

- In its simplest form, Multilink is a series of coloured blocks or cubes that can be joined together to form building blocks. The cubes are an ideal size for small hands to grab and fix, helping improve fine motor skills.What's more, each Multilink cube can be connected on every side, giving additional opportunities to build and explore upwards and sideways.
- Multilink has a wide variety of uses within the classroom including number, proportion and ratio,
 geometry, pattern, sequences, algebra, statistics and probability.


## BEAD STRINGS

- A bead string is a useful teaching resource for maths, that is best for assisting and engaging visual learners when it comes to maths. It is a string with coloured beads on it, that allows you to count and to solve maths problems visually, by moving the beads along the string. Bead strings are very similar to an abacus. So you could say that counting beads' name is also an Abacus too.
- Bead strings are ideal for helping to maintain student interaction in the classroom. They are just great for emergent learners during maths lessons.
Bead strings are perfect for use in both group work and individual led work. They are good for encouraging problem-solving, number
 exploration, and allowing for you to gain more familiarity with working with multiples of ten.
- Numicon gives pupils an idea of what the value of each number looks like and that when we count in ones, the number gets bigger by the same amount each time.
- In the case of Numicon, we add an additional hole every time. It appeals to children's visual understanding and their sense of pattern.
- Numbers are a very abstract concept and, as such, pupils can often feel overwhelmed when first learning maths. This can lead to anxiety around the subject throughout their lives.
- By having a physical material that they can hold and manipulate, pupils are more likely to be able to make the connections that allow them to work with numbers in an abstract form.
- Numicon is a teaching resource developed by Oxford University Press that comprises a number of plastic shapes that represent the numbers $\mathrm{I}-10$ and are relative in size to one another.
- Numicon comprises different shapes representing the numbers I-IO.
- As you can see from this image, the Numicon 2 is the same size as two Numicon I pieces, and so on. This allows pupils to understand the relationships between numbers.



## CUISENAIRE RODS

- Cuisenaire Rods are one of the most versatile manipulatives.A set consists of a collection of rectangular rods of 10 colors and naturally, each of those colors is a specific length. The list below (from shortest to longest) is a breakdown of each rod.
- White $=1$ centimeter long
- Red $=2 \mathrm{~cm}$ long
- Light Green $=3 \mathrm{~cm}$ long
- Purple $=4 \mathrm{~cm}$ long
- Yellow $=5 \mathrm{~cm}$ long
- Dark Green $=6 \mathrm{~cm}$ long
- Black $=7 \mathrm{~cm}$ long
- Brown $=8 \mathrm{~cm}$ long
- Blue $=9 \mathrm{~cm}$ long
- Orange $=10 \mathrm{~cm}$ long

There are no predetermined values of the rods, which is what makes them so useful to a range of different strands of the maths curriculum.


## APPLICATION

Using the manipulatives, show how many different ways that you could make 7.

Remember to discuss with your table WHY you are choosing the manipulative.

Use the Numicon to explain how you know that 7 is an odd number

How could you use the Numicon to show that an odd number plus an odd number will ALWAYS equal an even number

Find all of the multiples of 4 to 40 , using Numicon

## APPLICATION

Using the manipulatives, demonstrate how you could solve this calculation: $7+3$ =


## WITH THAT IN MIND...

Now use the manipulatives to solve/represent these calculations

- 14 + 16 =
- $56+44=$
- $100-\ldots=80$
- 356 + 28 =
- 472 - $81=$

Would you use the same choice for all calculations?

## MULTIPLICATION AND DIVISION

## Choose a manipulative

 to solve:$3 \times 4=$
$12 \div 4=$

## MULTIPLICATION

How could you use the manipulatives to solve these calculations?
$10 \times 6=$
$4^{2}=$
$23 \times 4=$
$32 \div 8=$

## MULTIPLICATIVE THINKING AND RECIPROCALS


-Manipulatives can be used to visually represent times tables and the related division facts.
-These are an example of bar models and later, these could be used as the foundations of fraction walls.
-Try representing :
$1 \times 10=10$
$\ldots \times \ldots=10$
$\ldots \times \quad=10$
$\ldots \times \quad=10$

## CUISENAIRE INVESTIGATION

I've been playing around with Cuisenaire rods, this is what I came to.
There are five different ways for us to make the pink rod using just red and white rods.

We count white, white, red as different from the white, red, white even though they both use two white rods and one red rod.



Using your Cuisenaire rods, can you work out how many different ways there are, using only the red and white rods, to make up:

- The light green rod?
- The yellow rod?
- The dark green rod?
- The black rod


## THE BEST BIT

- Most of these manipulatives don't have an assigned value; therefore, they can be used for anything!
- Look at these questions...
- Use the Cuisenaire rods to help

If blue represents 18 , what are the values of the pink rod and the yellow rod?
$\square$
If blue now represents 36 , what are the values for the pink and the yellow rods?
If blue represents 378 ...

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## NOW A CHALLENGE - ALGEBRA

- Algebra (finding unknown values) is an abstract concept, but children are taught (secretly) how to do this from an early stage in their school life.
- e.g $16+$ $\qquad$ $=20$
- This would like slightly different in a KS2 classroom, where the missing values are substituted for letters.
- e.g $16+\mathrm{a}=20$
- The calculations may also include the application of other areas such as BIDMAS
- e.g $16+2 a=20$
- All of this can be supported with manipulatives.


## SHOW SOLVING THIS WITH ANY OF THE MANIPULATIVES

- $16+\ldots=20$
- $16+a=20$
- $16+2 \mathrm{a}=20$


## YEAR 6 EXAMPLE OF MANIPULATIVES

- When children come across writing algebraic expressions, Cuisenaire rods can be used so that students can work from a position of familiarity. They may also see this as a pictorial representation as the bar model.
- Even if children aren't confident in representing missing values as letters, the use of manipulatives can work effectively to link the concrete world of number to the abstract world of algebra.


In this example 3 purple rods $(p)$ are equal to 2 greens (g) and a missing value.

Algebraic expression:
$2 g+x=3 p$

## YEAR 6 EXAMPLE OF MANIPULATIVES



- What could the missing value be?
- Is there more than one choice?
- Would this change the Algebraic expression?
(substitute missing
values for colours)

$$
\begin{aligned}
& 2 g+d=3 p \\
& 2 g+3 r=3 p \\
& 2 g+6 w=3 p
\end{aligned}
$$

Discussion: why would $2 g+2 g=3 p$ not be an accurate expression?

## FEEDBACK/QUESTIONS

